**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2018**

**Calculator-assumed**

# Marking Key

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The release date for this exam and marking scheme is

* **the end of week 8 of term 2, 2018**

**Question 8 (5 marks)**

|  |
| --- |
| Solution |
| If  then if  we have. Since we have assumed that we reject the negative square root.If then Since we have assumed that we reject the positive root.Hence the solution is  or . |
| Mathematical behaviours | Marks |
| * calculates the correct root assuming that (2 marks)
* calculates the correct root assuming that  (2 marks)
* states the overall solution correctly
 | 11 1 |

**Question 9 (a) (3 marks)**

|  |
| --- |
| Solution |
|  From the graph it appears that  has a zero at We evaluate  So by the factor theorem  is a factor of    |
| Mathematical behaviours | Marks |
| * obtains  as a possible zero of *p*
* demonstrates that
* deduces that  is a factor of
 | 111 |

**Question 9(b) (3 marks)**

|  |
| --- |
| Solution |
| Long division gives that  If  by either using the quadratic formula or completing the squareHence  and  are the conjugate linear factors of    |
| Mathematical behaviours | Marks |
| * identifies the correct quadratic factor
* determines correctly the zeros of the quadratic factor
* states correctly the corresponding conjugate linear factors
 | 11 1 |

**Question 10(a) (2 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * correctly states the horizontal and vertical components of
* simplifies correctly
 | 11 |

**Question 10(b) (2 marks)**

|  |
| --- |
| Solution |
| The acceleration is  |
| Mathematical behaviours | Marks |
| * integrates  to determine
* determines the constant and states the correct
 | 11 |

**Question 10(c) (6 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * integrates  to determine  (accept assumption of constants of integration as 0 from initial conditions, without statement)
* uses vertical component of  determine the time when max height is reached
* determines the max height
* recognises that the max horizontal distance when vertical component =0
* determines the time when max distance is reached
* determines max distance
 | 111111 |

**Question 11 (a) (2 marks)**

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| --- |
| Solution |
| Now  and  Since a pair of adjacent sides in the parallelogram *P* have the same length, all sides have the same length.Hence Pis a rhombus. |
| Mathematical behaviours | Marks |
| * calculates the correct values of  and
* makes a valid deduction about the lengths of all sides of P
 | 11 |

**Question 11 (b) (3 marks)**

|  |
| --- |
| Solution |
| Denote  . Then  Denote  . Then  Hence  So the required angle is 2.21 radians (correct to 2 decimal places)  |
| Mathematical behaviours | Marks |
| * determines correctly the value of
* determines correctly the value
* gives the required answer to the prescribed accuracy
 | 1 1 1 |

**Question 11 (c) (2 marks)**

|  |
| --- |
| Solution |
| We see that  and that  Hence it follows that  |
| Mathematical behaviours | Marks |
| * evaluates correctly the value of
* evaluates correctly the value of .
 | 11 |

**Question 11 (d) (4 marks)**

|  |
| --- |
| Solution |
| The diagonals of P are  and  .From part (c) we deduce that …….(i)Hence one diagonal is twice as long as the other. Also we have that  ………(ii)Hence the angle between the diagonals is  .  |
| Mathematical behaviours | Marks |
| * derives the result (i)
* deduces the correct ratio of the lengths of the diagonals
* derives the result (ii)
* deduces the correct angle between the diagonals
 | 1111 |

**Question 12(a) (3 marks)**

|  |
| --- |
| Solution |
| Since the function involves a square root we need to check that the quadratic is positive for all real values of.As   the function is defined for all real values |
| Mathematical behaviours | Marks |
| * notes that we need to show that the contents of the square root is positive
* either completes the square or shows the quadratic never equals zero
* deduces the required result
 | 1 1 1 |

**Question 12(b) (3 marks)**

|  |
| --- |
| Solution |
| If  then  This then implies that  or   |
| Mathematical behaviours | Marks |
| * forms an equation without the square root expressing the fact that
* solves the equation
* states the two possible solutions
 | 1 1 1 |

**Question 12(c) (3 marks)**

|  |
| --- |
| Solution |
| The function is not one-to-one. If the function were 1-1 then would force .Since here we have the possibility that  we do not have a 1-1 function |
| Mathematical behaviours | Marks |
| * states the correct conclusion
* indicates the property that must be satisfied by a 1-1 function
* justifies why this function does not possess this property
 | 11 1 |

**Question 13 (6 marks)**

|  |
| --- |
| Solution |
| If  then  .Also so that  for  . Hence  or  . Written in polar form,  cis with  or  .Moreover, cis cis cis cisand  so the solutions are. ,  and . |
|  Mathematical behaviours | Marks |
|  * derives the result that
* deduces the correct value of
* obtains one value of
* determines all four possible values of
* writes down the the real and imaginary parts of the four ‘‘cis’ values
* solves correctly for the real and imaginary parts of all four solutions
 | 111111 |

**Question 14(a) (3 marks)**

|  |
| --- |
| Solution |
| If  Therefore the inverse function is (Alternatively students may define the other branch with  so  .) |
| Mathematical behaviours | Marks |
| * writes the equation
* solves for
* states correct inverse function
 | 11 1 |

**Question 14(b) (2 marks)**

|  |
| --- |
| Solution |
| Domain is  Range is (If student took the alternative definition the range becomes  .) |
| Mathematical behaviours | Marks |
| * states correct domain
* states correct range
 | 11 |

**Question 14(c) (2 marks)**

|  |
| --- |
| Solution |
|    |
| Mathematical behaviours | Marks |
| * gives neat sketch of
* gives neat sketch of
 | 11 |

**Question 14(d) (2 marks)**

|  |
| --- |
| Solution |
| The two graphs are reflections of each other in the line   |
| Mathematical behaviours | Marks |
| States the correct geometrical relationship with marks * for mentioning reflection
* for giving the equation of the line of reflection
 |  1 1 |

**Question 15(a)(b) (3+2 marks)**

|  |
| --- |
| Solution |
|  since  Point  moves in a horizontal circle, +2 units above the as indicated by the 2.The centre of the circle is (0,0,2) and the radius is 1 unit.At  so the particle starts at (1,0,2) and at  so the particle moves in an anticlockwise direction.  |
| Mathematical behaviours | Marks |
| * indicates that the particle moves in plane parallel to the and 2 units above it
* indicates that it is a circle
* states the centre and radius of the circle
* indicates that the particle moves in an anticlockwise direction
* provides an appropriately labelled diagram of the circle in roughly the right position
 | 11111 |

**Question 15(c) (2 marks)**

|  |
| --- |
| Solution |
|  The particle remains at a constant distance from the origin. |
| Mathematical behaviours | Marks |
| * determines the correct distance
* states that the distance from the origin is constant over time
 | 11 |

**Question 15(d) (2 marks)**

|  |
| --- |
| Solution |
| The position vector is given by $p\left(t\right)=\cos(\left(2t\right))i+sin\left(2t\right)j+2k$Differentiating   |
| Mathematical behaviours | Marks |
| * determines the correct velocity
* (allow one mark if attempts to differentiate but makes an error)
 | 2 |

**Question 15(e) (2 marks)**

|  |
| --- |
| Solution |
| Given  moves in a circle in the  , with centre (0,2) and radius 2, the vector equation is  with  |
| Mathematical behaviours | Marks |
| * states the correct cartesian equation
* remembers to state that
 | 11 |

**Question 15(f) (3 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * indicates the need to determine
* determines
* calculates the correct distance
 | 111 |

**Question 16(a) (4 marks)**

|  |
| --- |
| Solution |
| By De Moivre’s theorem we have  .Expanding and taking real parts gives        |
| Mathematical behaviours | Marks |
| * uses De Moivre’s theorem appropriately
* expands the fourth power of the expression correctly
* takes the real parts of each side
* replaces  and simplifies to obtain the result
 | 1111 |

**Question 16 (b) (5 marks)**

|  |
| --- |
| Solution |
| Let  where  . Then by part (a) we have  Hence the maximum and minimum values of  are  .At maximum values  so that  or  within the range.Since we have .At minimum values  so that  or  within range whence  . In summary, the maximum value of $p(x)$ is 1, and occurs at $x=1,0 or-1$ and the minimum value of $p(x)$ is -1, and occurs at $x=\pm 1/\sqrt{2}$ |
|  Mathematical behaviours | Marks |
| * derives 1 and -1 as the extreme values of
* determines the correct values of θ at the maximum
* infers the corresponding correct values of
* determines the correct values of θ at the minimum
* infers the corresponding correct values of
 | 11111 |

**Question 17(a) (4 marks)**

|  |
| --- |
| Solution |
| We have  . Domain is  and range is  (all real numbers)Similarly . Domain is  and range is (all real numbers) |
| Mathematical behaviours | Marks |
| * determines the two composite functions correctly (one mark for each)
* states domain and range of
* states domain and range of
 | 11 1 |

**Question 17(b) (2 marks)**

|  |
| --- |
| Solution |
| If  then  |
| Mathematical behaviours | Marks |
| * forms an appropriate equation for
* solves correctly
 | 11  |

**Question 18(a) (1 mark)**

|  |
| --- |
| Solution |
| Since plane  is parallel to the , and  everywhere on the plane the equation is  (in vector form the equation is **r.i****)** |
| Mathematical behaviours | Marks |
| * states the correct equation of the plane
 | 1 |

**Question 18(b) (3 marks)**

|  |
| --- |
| Solution |
| Largest sphere will have a diameter = 3 units, so the radius = 1.5 units and centre = Vector form:  Cartesian form:   |
| Mathematical behaviours | Marks |
| * states the centre and radius of the sphere
* states the vector equation
* states the Cartesian equation
 | 111 |

**Question 18(c) (3 marks)**

|  |
| --- |
| Solution |
|    |
| Mathematical behaviours | Marks |
| * determines the cross product of two appropriate vectors
* uses the cross product to determine the equation of the plane
* states the correct plane equation
 | 111 |

**Question 18(d) (5 marks)**

|  |
| --- |
| Solution |
| 1.
2. Substituting into

 so its position vector is **i**+**j** + **k**  |
| Mathematical behaviours | Marks |
| * determines the position vector for
* uses the position vectors of  and to determine the vector equation of the line
* states the equation of the line in equivalent parametric form.
* substitutes into the equation of the plane

(or whatever found in part (c))* states the position vector of the point of intersection
 | 11111 |

**Question 18(e) (3 marks)**

|  |
| --- |
| Solution |
|   Using the dot product of and  or the angle between vectors on a CAS calculator the angle between the line and the plane is 91.6°. |
| Mathematical behaviours | Marks |
| * determines an appropriate vector in the line (e.g )
* determines an appropriate vector in the plane (e.g)
* states the angle between the plane and the line (accept any suitable rounding)
 | 111 |